Department of Mathematics

Comprehensive Examination–Option I

2016 Autumn

Algebra

1. Prove: Each subgroup of a cyclic group is cyclic.

2. Suppose that : → is a ring homomorphism from a commutative ring with unity 1 onto a ring . Prove:
   (a) ker is an ideal of .
   (b) is a commutative ring with unity.

3. Let Q[ ] be the ring of polynomials with coe□cients in Q, the field of all rational numbers. Prove or disprove:
   (a) (+ 4 + 3 + 2) is a maximal ideal of Q[ ].
   (b) Q[ ] (+ 4 + 3 + 2) is a field.

4. Let A be a real × matrix and let A^T be its transpose matrix. Prove that A^TA is invertible if and only if the column vectors of A are linearly independent

Department of Mathematics

Comprehensive Examination–Option I

2016 Autumn

Complex Analysis

1.

2.

3.

4.

Department of Mathematics

Comprehensive Examination–Option I

2016 Autumn

Real Analysis

1. **Let A = { √– | □ Q and 0 ≤ }. Prove that A has zero measure (with the standard measure of Euclidean space).**

   Hint: Use the fact that the measure of a countable disjoint un

Department of Mathematics

Comprehensive Examination–Option I

2016 Autumn

Topology

1. **Prove: Each metric space is Hausdor.**

2. **Suppose and are topological spaces, : → is continuous, and** A

Department of Mathematics

Department of Mathematics

Comprehensive Examination–Option III

2016 Autumn

Numerical Analysis

1. (a) Prove that there exists exactly one solution of the equation arctan = $^{-x}$.

   (b) Use Newton's method to find an approximation of the solution such that | − | 10^{-} .

   (c) Prove that your approximation is in fact within 10^{-} of (the exact) .

   Note: For this problem you may not use any graphing or root finding capabilities of your calculator.

2. Consider the following di□erence formula.

$$'(\quad) = \frac{(\quad + 4\ ) - 12\ (\quad + 2\ ) + 32\ (\quad +\ ) - 21\ (\ )}{12} - \quad (\ ) \quad + \quad (\ )$$

   Note that $\frac{(\quad + 4\ ) - 12\ (\quad + 2\ ) + 32\ (\quad +\ ) - 21\ (\ )}{12}$ is a ( ) approximation of $'(\quad)$.
   Use the formula to construct a ( ) approximation of $'(\quad)$ that involves

$$(\ ) \quad (\quad +\ ) \quad (\quad + 2\ ) \quad (\quad + 4\ ) \text{ and } (\quad + 8\ )$$

3. Let A be a symmetric pentadiagonal positive definite matrix of the following form.

$$\left(\qquad L \right.$$

Department of Mathematics

Comprehensive Examination–Option III

2016 Autumn

Linear Programming

1. Solve the following linear programming problem using the Primal Simplex Method (NOT the Dual Method).

$$
\begin{array}{rllll}
\text{minimize} & 6 & +3 & +4 & \\
\text{subject to} & 3 & + & +3 & \geq 9 \\
 & 2 & - & +4 & \leq 8 \\
 & 2 & +2 & + & \geq 9 \\
 & & & {}_j & \geq 0
\end{array}
$$

2. Consider the following maximization problem.

$$
\begin{array}{rllll}
\text{maximize} & 40 & +20 & +10 & \\
\text{subject to} & 3 & + & +\ 4 & \leq 18 \\
 & 2 & +\ 2 & + & \leq 16 \\
 & & + & +\ 3 & = 14 \\
 & & & & \geq\ 0
\end{array}
$$

The beginning and final tableaux in the Simplex method are given in the following table.

	3	1	4	1	0	0	18
	2	2	1	0	1	0	16
	1	1	3	0	0	1	14
	−40	−20	−10	0	0	0	0

Linear Programming — continued

3. Using the Complementary Slackness Theorem prove or disprove the following statement: (4 2 0) is the optimal solution of the maximization problem below.

maximize	5	+ 2	+		
subject to		+ 2	+ 2		≤ 14
	2	+ 3	+ 4		≤ 14
		+ 2	+		= 8
				$_j$	≥ 0

4. Solve the following transportation problem, with transportation costs given inside the table. The supplies are listed along the left, and the demands are listed along the top.
Make sure to give the final minimum cost.

	30	20	25	30	25
20	7	11	10	9	8
30	7	3	2	4	5
30	6	6	9	4	5
50	8	10	12	11	9

Department of Mathematics

Comprehensive Examination–Option III

2016 Autumn

Probability

1. A coin is biased so that heads is twice as likely as tails. For three independent tosses of the coin find
   (a) the probability distribution ( ) of , the total number of heads;
   (b) the probability of getting at most two heads;
   (c) the distribution function ( ) for ;
   (d) (2) using part (c);
   (e) the expected number of heads, ( ).

2. Let be a Gamma random variable with parameters and , where 0 and 0. The Gamma